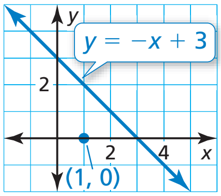
Finding the Distance from a Point to a Line

Recall that the distance from a point to a line is the length of the perpendicular segment from the point to the line.

EXAMPLE 5 Finding the Distance from a Point to a Line



Find the distance from the point (1, 0) to the line $y = -x + 3$.

SOLUTION

Step 1 Find an equation of the line perpendicular to the line $y = -x + 3$ that passes through the point (1, 0).

First, find the slope m of the perpendicular line. The line $y = -x + 3$ has a slope of -1. Use the Slopes of Perpendicular Lines Theorem.

$-1 \cdot m = -1$	The product of the slopes of ⊥ lines is −1.
$m = 1$	Divide each side by −1.

Then find the y-intercept b by using $m = 1$ and $(x, y) = (1, 0)$.

$y = mx + b$	Use slope-intercept form.
$0 = 1(1) + b$	Substitute for x, y, and m.
$-1 = b$	Solve for b.

Because $m = 1$ and $b = -1$, an equation of the line is $y = x - 1$.

REMEMBER

Recall that the solution of a system of two linear equations in two variables gives the coordinates of the point of intersection of the graphs of the equations.

There are two special cases when the lines have the same slope.

- When the system has no solution, the lines are parallel.
- When the system has infinitely many solutions, the lines coincide.

Step 2 Use the two equations to write and solve a system of equations to find the point where the two lines intersect.

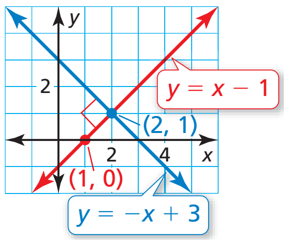
$y = -x + 3$	Equation 1
$y = x - 1$	Equation 2

Substitute $-x + 3$ for y in Equation 2.

$y = x - 1$	Equation 2
$-x + 3 = x - 1$	Substitute $-x + 3$ for y.
$x = 2$	Solve for x.

Substitute 2 for x in Equation 1 and solve for y.

$y = -x + 3$	Equation 1
$y = -2 + 3$	Substitute 2 for x.
$y = 1$	Simplify.



So, the perpendicular lines intersect at (2, 1).

Step 3 Use the Distance Formula to find the distance from (1, 0) to (2, 1).

$$\text{distance} = \sqrt{(1-2)^2 + (0-1)^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \approx 1.4$$

▶ So, the distance from the point (1, 0) to the line $y = -x + 3$ is about 1.4 units.