Geometry Info Sheet #36

Right Triangle Similarity and Geometric Mean

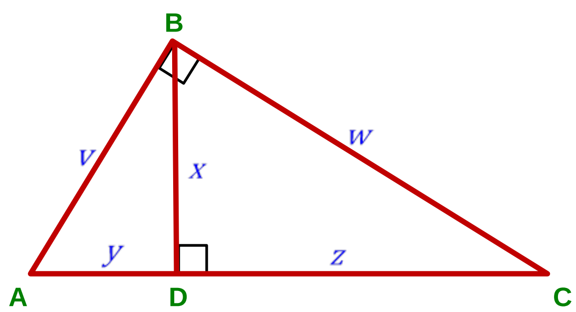
Definitions

Geometric Mean: The n^{th} root of the product of n numbers

The geometric mean of two positive numbers, a and b, is $\sqrt{ab}$, and the geometric mean of three positive numbers, a, b, and c, is $\sqrt[3]{abc}$, and the geometric mean of four positive numbers, a, b, c, and d, is $\sqrt[4]{abcd}$, and so on.

Theorems

Hypotenuse Altitude Theorem: If an altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\Delta ABC \sim \Delta ADB \sim \Delta BDC$

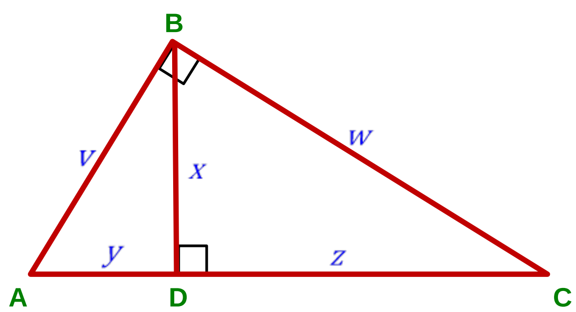
Proportion Examples

$\frac{v}{w} = \frac{y}{x} = \frac{x}{z}$ or $\frac{w}{y+z} = \frac{x}{v} = \frac{z}{w}$ or $\frac{v}{y+z} = \frac{y}{v} = \frac{x}{w}$

Corollaries to Hypotenuse Altitude Theorem

In a right triangle, the length of the altitude to the hypotenuse is the geometric mean of the lengths of the segments into which it divides the hypotenuse.

In a right triangle, the altitude to the hypotenuse divides the hypotenuse so that the length of each leg is the geometric mean of the lengths of the hypotenuse and the adjacent hypotenuse segment.



$x^2 = yz \longrightarrow x = \sqrt{yz}$

$v^2 = y(y+z) \longrightarrow v = \sqrt{y(y+z)}$

$w^2 = z(y+z) \longrightarrow w = \sqrt{z(y+z)}$