Geometry Info Sheet #38

Trigonometry with Oblique Triangles

Triangle Area Formulas

Base and Height Known:

The area A of any triangle with base b and height h is given by: $A = \frac{1}{2}bh$

All Three Sides Known:

For any ΔXYZ with sides x, y, and z, and with semiperimeter (half of the perimeter) s, the area A is given by: $A = \sqrt{s(s-x)(s-y)(s-z)}$

This is called **Heron's Formula**, and it can be used when all three sides of the triangle are known.

Two Sides and Included Angle Known:

For any ΔXYZ with sides x, y, and z, depending on which sides and which angle are known, the area A is given by:
$A = \frac{1}{2}xy(\sin Z)$ **or** $A = \frac{1}{2}yz(\sin X)$ **or** $A = \frac{1}{2}xz(\sin Y)$

Note that for these formulas to work, the known angle must be between the two known sides of the triangle.

Law of Sines

For any ΔABC with sides a, b, and c: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

The **Law of Sines** can be used to find the missing angles and sides in any triangle when the following information is already known: 1) two angles and any side **or** 2) two sides and a non-included angle

When using the Law of Sines to find all angles and sides for a triangle in which two sides and a non-included angle are known, if the known side opposite the known angle is shorter than the known adjacent side, then there will be two different answers (qualifying triangles). This situation is known as the **ambiguous case** of the Law of Sines.

Law of Cosines

For any ΔABC with sides a, b, and c:

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

The **Law of Cosines** can be used to find the missing angles and sides in any triangle when the following information is already known: 1) two sides and the included angle **or** 2) all three sides