Geometry Info Sheet #50

Prisms: Diagonals, Surface Area, Lateral Area, and Volume

Definitions

Prism: A polyhedron with two congruent and parallel n-sided polygonal faces (its bases); it has n other faces (sides) that connect the corresponding edges of the two bases; a prism has no curves (only flat surfaces) and is named by the shape of its bases

A prism's two congruent and parallel n-sided polygons are its **bases**. Any side of a prism that is not a base is a parallelogram known as a **lateral face**. The edges of the lateral faces that are not also edges of a base are the **lateral edges**. An **altitude** of a prism is a segment with endpoints in the planes containing the two bases and perpendicular to both. The length of an altitude is its **height**.

In a **right prism**, the lateral faces are all rectangles, the bases are directly opposite each other, and every lateral edge is an altitude. Unlike an **oblique prism**, which "leans" to one side and has at least one non-rectangular lateral face, a right prism does not "lean" at all.

A **right rectangular prism** is a box-shaped figure in which all six faces are rectangles and every angle is a right angle. A **cube** is a right rectangular prism in which all six faces are squares and all three dimensions (length, width, and height) are equal.

Diagonal: In a right rectangular prism, a segment joining opposite corners of the prism and cutting through the interior space of the prism; every right rectangular prism has four diagonals

Cavalieri's Principle: If two solids have the same height and same cross-sectional area at every level, then they have the same volume

Formulas

The **length** of a **diagonal** d in a **right rectangular prism** with dimensions length ℓ, width w, and height h is given by: $d = \sqrt{\ell^2 + w^2 + h^2}$

The **surface area** S, and **volume** V, of a **right rectangular prism** with dimensions length ℓ, width w, and height h are given by: $S = 2\ell w + 2wh + 2\ell h$, $V = \ell wh$

The **surface area** S, and **volume** V, of a **cube** with side s are given by: $S = 6s^2$, $V = s^3$

The **lateral area** L of **any right prism** with perimeter p and height h is given by: $L = ph$

The **surface area** S of **any right prism** with base area B, lateral area L, perimeter p, and height h is given by: $S = 2B + L$ **or** $S = 2B + ph$

The **volume** V of **any prism** with base area B and height h is given by: $V = Bh$